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Supplemental Material for “Parallel Computation of 3D Morse-Smale complexes”

Nithin Shivashankar [1] and Vijay Natarajan [1,2]

[1]Department of Computer Science and Automation, Indian Institue of Science, Bangalore-12, India
[2]Supercomputer Education and Research Center, Indian Institue of Science, Bangalore-12, India

Appendix: Handling Large Data

The following section discusses a divide and conquer technique to compute the Morse-Smale(MS) complex of large 3D datasets defined on structured grids with the assumption that they do not fit in memory. This method is based on the technique for large 2D datasets discussed in [SMN11]. The following section repeats the relevant parts from the 2D technique.

1. Out-of-core algorithm

The computation of the MS complex of 3D scalar functions defined on large structured grids that do not fit in memory is done in five stages (see Figure 1).

- Split dataset
- Compute MS complex on sub-domains
- Merge sub-domains
- Traverse merge history
- Extract geometry

Split Dataset. The data is first hierarchically partitioned into sub-domain blocks. The 3D structured grid domain is successively partitioned along each axis in an oct-tree fashion. The partitioning stops when the sub-domains are small enough to fit in memory. Exterior cells that are incident on shared boundaries are also included. In our experiments the sub-domains were $256 \times 256 \times 256$ blocks.

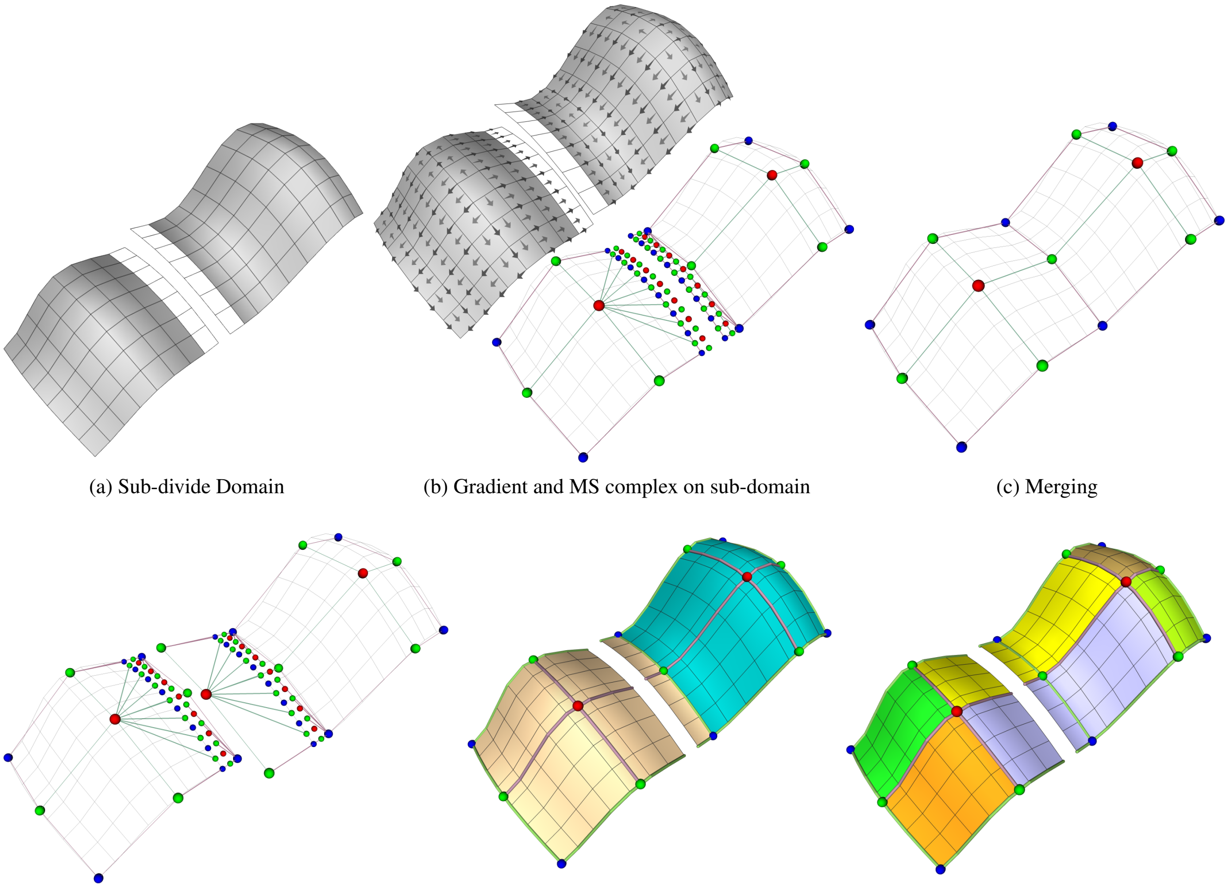
Gradient and MS Complex on sub-domains. Gradient pairs are computed within a sub-domain using Algorithm 1 and Algorithm 2 from section 4. The cell complex of the sub-domain is extended to include the set of cells that are incident on the shared boundary of sub-domains but gradient pairs are computed only on the initial sub-domain cell complex (see Figure 1b). Thus, we obtain identical pairings for cells along the shared boundary when we process all sub-domains that share the boundary cell. To facilitate merging we mark all gradient pairs that cross a shared boundary as critical (see Figure 1b).

The MS complex is simplified by repeated cancellation of critical pairs ordered based on topological persistence within sub-domains. This helps in the early elimination of noisy critical cells. To ensure combinatorial consistency of the MS complex after merging, we allow cancellation of pairs of critical cells only if (a) they are not already paired to be canceled when merged (b) neither among the pair is on a shared boundary (c) at most one among the pair has a connection to a critical cell that is paired to cross a shared boundary.

Merging sub-domain MS complexes. Next, the sub-domains are merged in a bottom up fashion by identifying and canceling boundary critical cell pairs along the common boundary between the sub-domains. The cancellations merges the MS complexes of the sub-domains resulting in the MS complex of the input function. As a consequence of the ORDER INDEPENDENT CANCELLATION LEMMA ([SMN11]), we can schedule cancellations of boundary critical cell pairs in any order. Thus the resulting MS complex is equivalent to the MS complex computed without splitting the domain. This MS complex can be further simplified by repeated cancellations of critical cell pairs.

History Tree. One of the implications of declaring all boundary cells and their outgoing / incoming pairs as critical is the creation of a large number of critical cells. Since the merge operation involves cancellation of critical cells, the ascending and descending manifolds need to be computed and merged. The number of cells that are present in the ascending / descending manifold of a critical cell is linear in the number cells in the cell complex. Storing the geometry of intermediate complexes results in a large memory footprint.

The boundary critical cells represent gateways through which flow enters / leaves a sub-domain. Therefore, recording the combinatorial connectivity to a surviving critical point at the boundary is sufficient to compute the ascend-



(a) Sub-divide Domain (b) Gradient and MS complex on sub-domain (c) Merging

(d) Traversing merge history (e) Extracting geometry

Figure 1: The MS complex for a large domain is computed in five stages.(a) Data is split into sub-domains. (b) Gradient pairs are computed on sub-domains. Unpaired cells and gradient pairs incident on shared boundary are marked critical. The combinatorial MS complex is computed for each sub-domain. (c) The combinatorial MS complex of the domain is computed by identifying and canceling gradient pairs that are incident on the shared boundary. (d) The history of merge cancellations is traversed to reveal the incidence of critical cells across sub-domains. This information is used to trace the geometry of the cells of the MS complex. (e) The geometry of the descending and ascending manifold of incident critical cells restricted to the sub-domain is extracted.

ing/descending manifold restricted to the sub-domain. We record this information during the merge step and are therefore able to compute the 1-skeleton of the MS complex with a small memory footprint. The recorded combinatorial connectivity between boundary critical points is used later to extract the geometry of the gradient paths. We now describe how to traverse the history of cancellations to compute the geometry of the arcs.

Consider a series of k cancellations to determine the combinatorial connection between two critical cells p^i and q^{i-1}. The series of canceled critical cell pairs is equal to the gradient path connecting the two critical cells:

$$p,\dots,\alpha^{i-1}_{k-1},\sigma^{i}_{k-1},\dots,\alpha^{i-1}_{k},\sigma^{i}_{k},\dots,\alpha^{i-1}_{k-2},\sigma^{i}_{k-2},\dots,q$$

Consider the final cancellation that determines the connection between p and q. Before cancellation, p is contained in ascending connections of α_k and q is contained in the descending connections of σ_k. Before the cancellation of the $(k-1)^{th}$ pair, α^{i-1}_k is connected to σ^i_{k-1}. By retaining this information, after the k^{th} cancellation we can infer that σ_{k-1} is connected to all surviving critical cells in the descending connections of α_k's pair. Extending this further to previous cancellations, we see that if we traverse the critical cell pairs in reverse order of their cancellations, we can infer the entire geometry of the gradient path. This is accomplished by traversing the history tree, which records all merges, in a top-down manner. At the leaf of the history tree, we obtain the combinatorial connections from the BFS traversal within the sub-domain.

Geometry extraction. The history tree traversal returns the points of entry and exit of all gradient paths entering or leaving the sub-domain. Thus the geometry of the descending/ascending manifold of a critical cell restricted to the sub-domain can be computed by tracking the gradient paths from the cells of entry/exit that are on shared boundaries. If the critical cell is contained within a sub-domain, then the geometry is computed as indicated in the first stage.

References

[SMN11] SHIVASHANKAR N., MAADASWAMY S., NATARAJAN V.: Parallel computation of 2D Morse-Smale complexes. *IEEE Transactions on Visualization and Computer Graphics 99*, PrePrints (2011). 1