An Introduction to Reinforcement Learning

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August 2014

What is Reinforcement Learning?

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[Video[1] of little girl learning to ride bicycle]

1. https://www.youtube.com/watch?v=Qv43pKlVZXk

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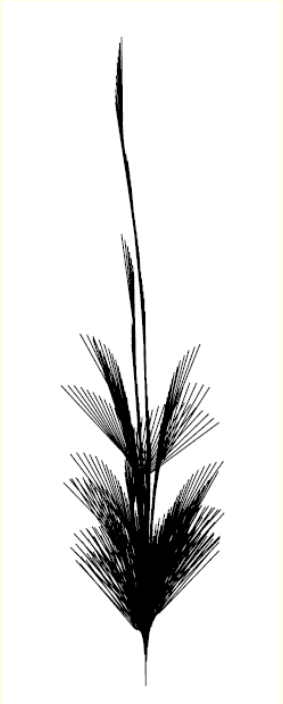

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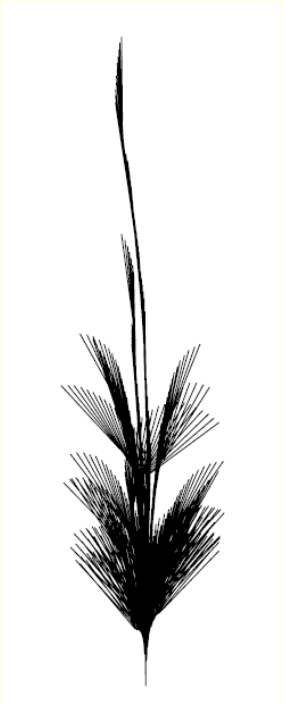

Learning to Drive a Bicycle using Reinforcement Learning and Shaping
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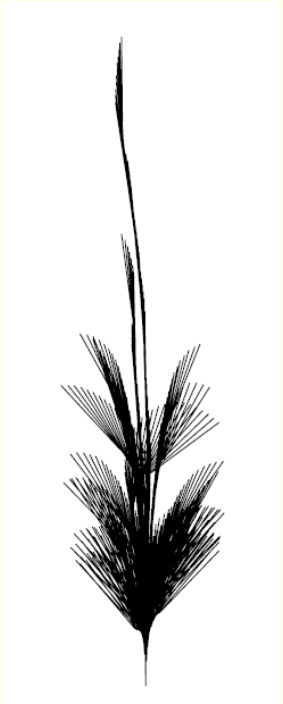

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Learning by trial and error to perform *sequential* decision making.

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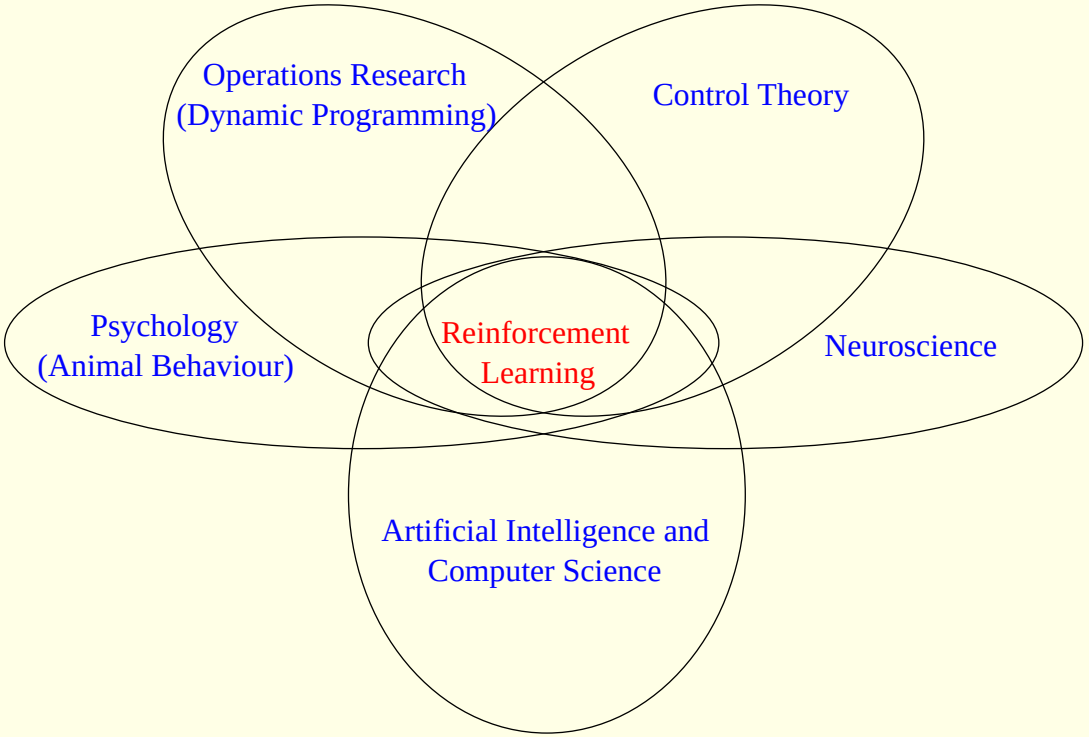
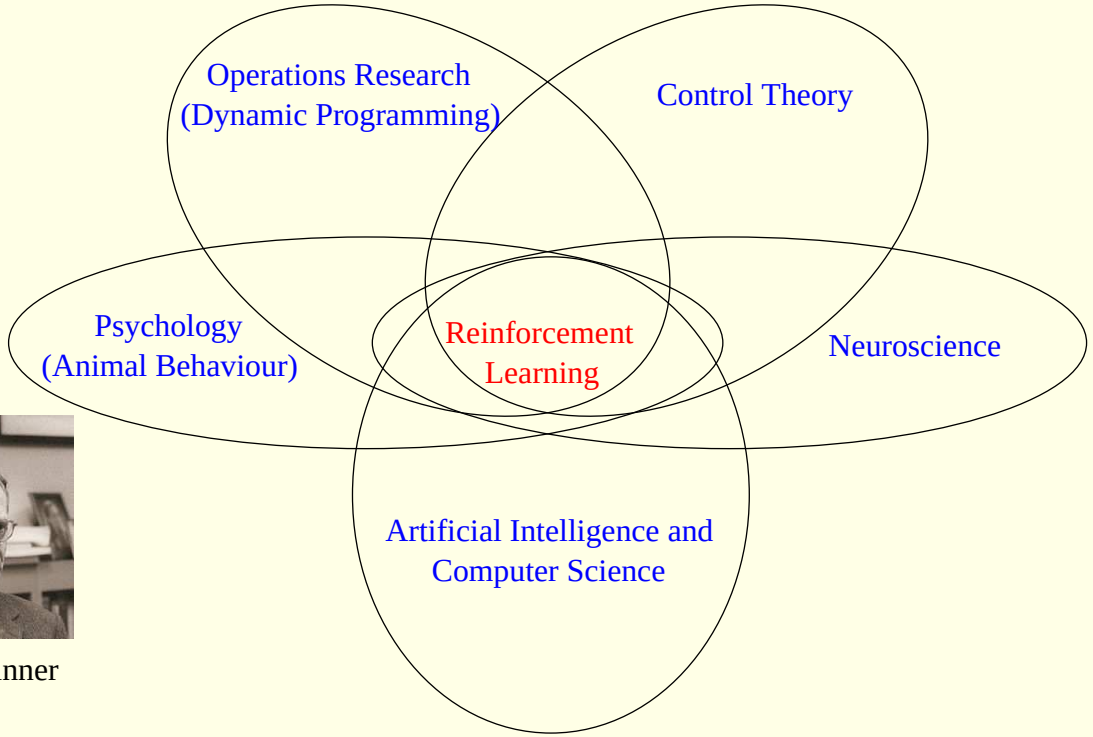
Our View of Reinforcement Learning



Our View of Reinforcement Learning

B. F. Skinner



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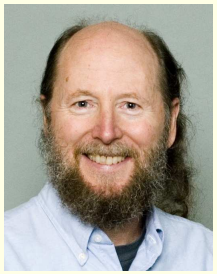
R. S. Sutton

Resources

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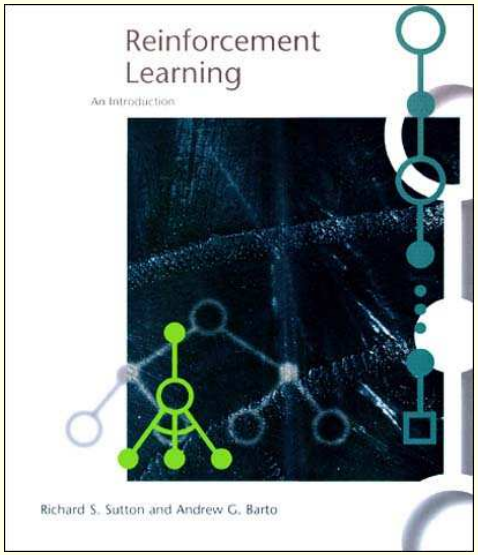

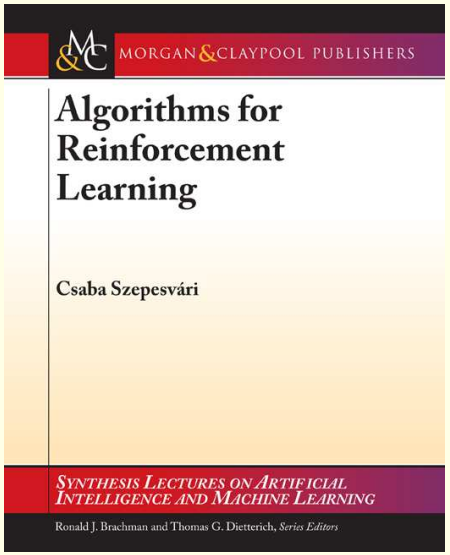

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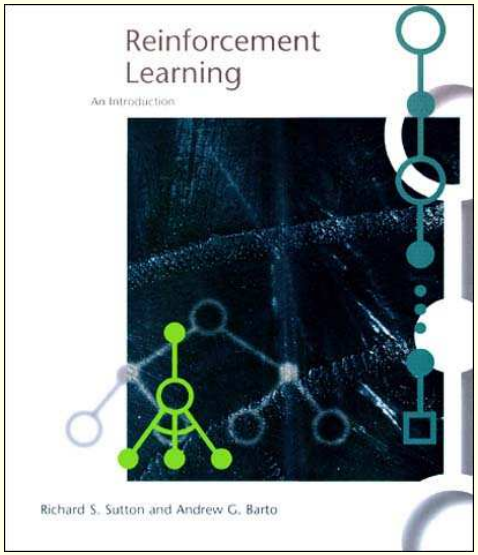

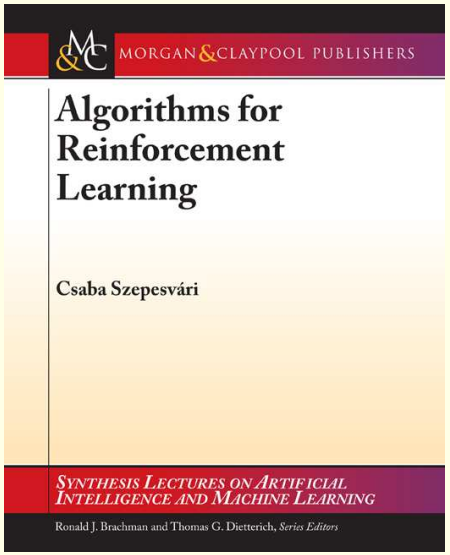

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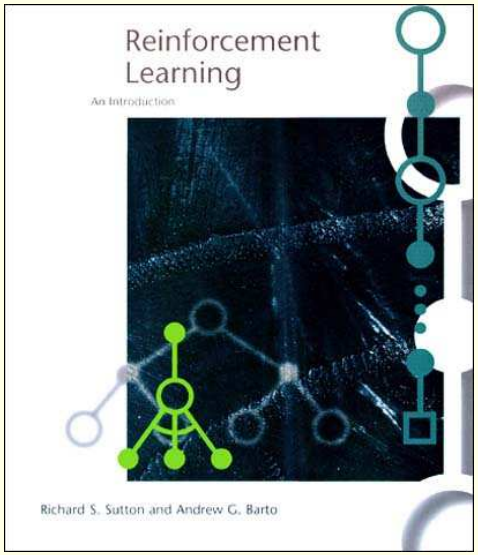

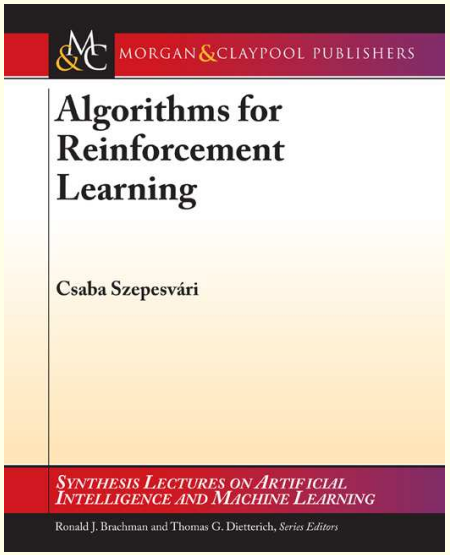

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Today's Class

1. Markov decision problems
2. Bellman's (Optimality) Equations, planning and learning
3. Challenges
4. Summary

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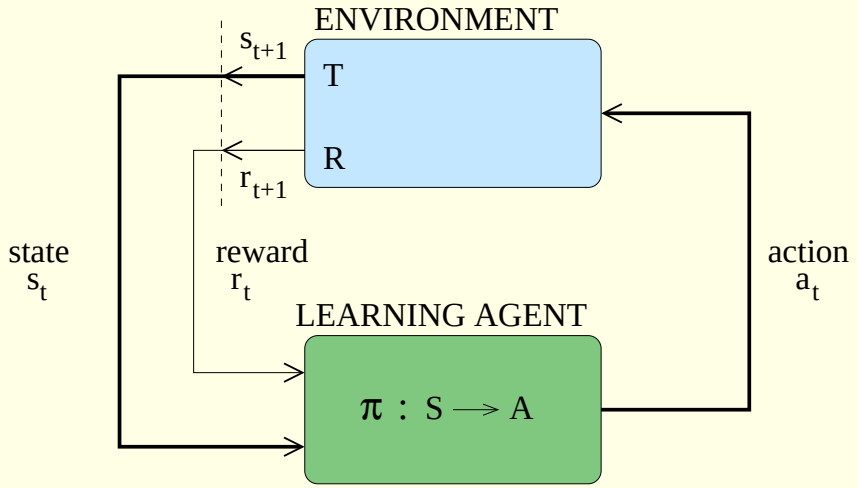

S: set of states.
A: set of actions.
T: transition function. $\forall s \in S, \forall a \in A$, $T(s, a)$ is a distribution over S.
R: reward function. $\forall s, s' \in S, \forall a \in A$, $R(s, a, s')$ is a finite real number.
γ: discount factor. $0 \leq \gamma < 1$.

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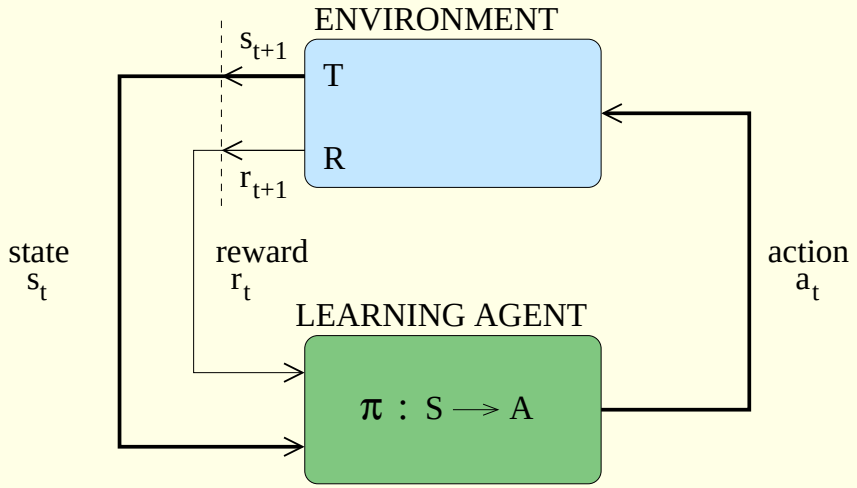

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Trajectory over time: $s_0, a_0, r_1, s_1, a_1, r_2, \ldots, s_t, a_t, r_{t+1}, s_{t+1}, \ldots$.

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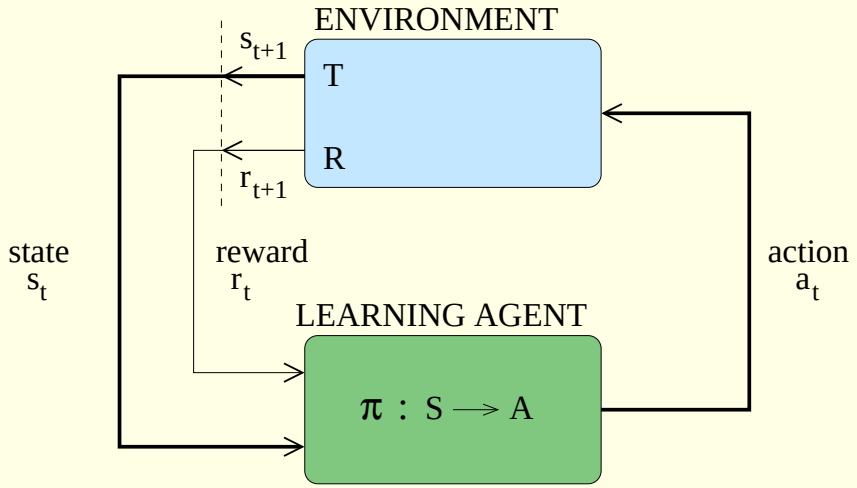

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Value, or expected long-term reward, of state s under policy π:

$$V^{\pi}(s) = \mathbb{E}[r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots \text{ to } \infty | s_0 = s, a_i = \pi(s_i)].$$

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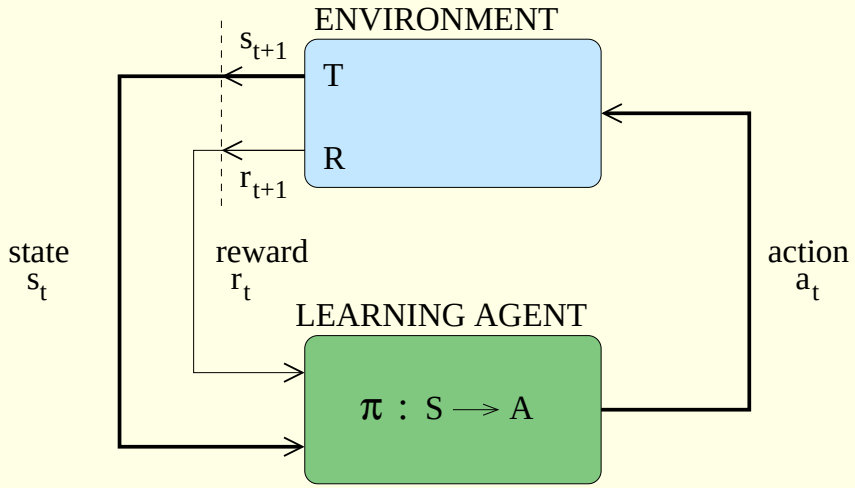

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Objective: "Find π such that $V^\pi(s)$ is maximal $\forall s \in S$."

Examples

What are the agent and environment? What are S, A, T, and R?

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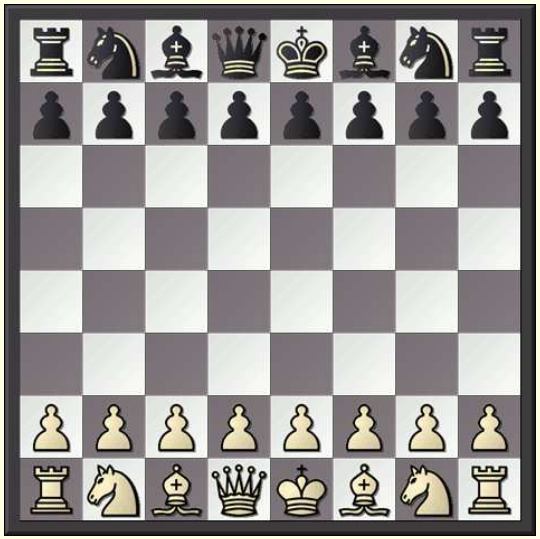

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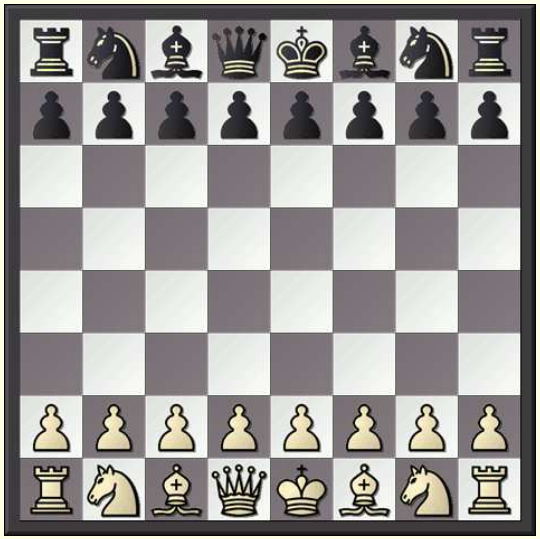

An Application of Reinforcement Learning to Aerobatic Helicopter Flight
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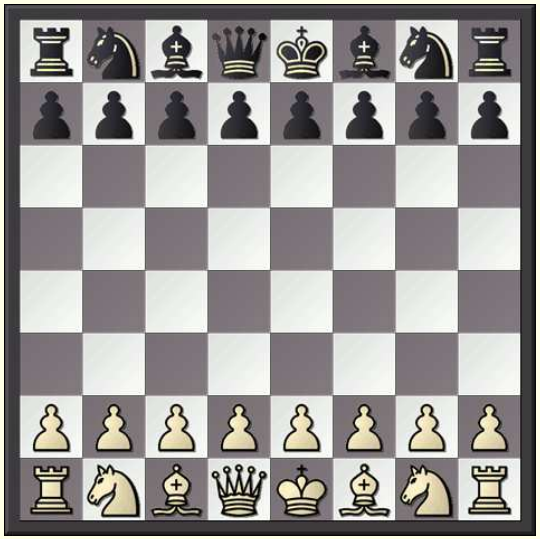

[Video[3] of Tetris]

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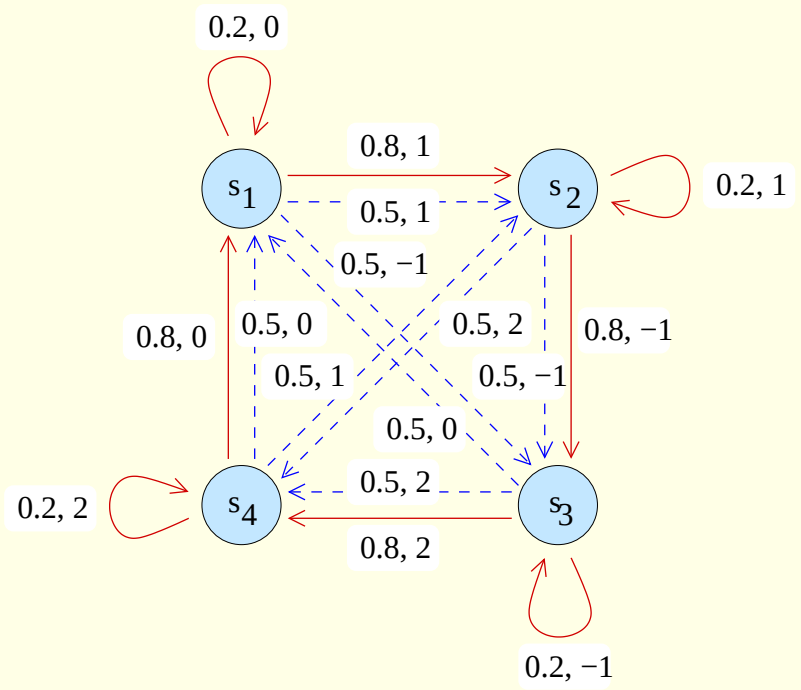
Illustration: MDPs as State Transition Diagrams



Notation: "transition probability, reward" marked on each arrow

States: s_1, s_2, s_3, and s_4.

Actions: Red (solid lines) and blue (dotted lines).

Transitions: Red action leads to same state with 20% chance, to next-clockwise state with 80% chance. Blue action leads to next-clockwise state or 2-removed-clockwise state with equal (50%) probability.

Rewards: $R(*, *, s_1) = 0$, $R(*, *, s_2) = 1$, $R(*, *, s_3) = -1$, $R(*, *, s_4) = 2$.

Discount factor: $\gamma = 0.9$.

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Bellman's Equations

Recall that

$$V^{\pi}(s) = \mathbb{E}[r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots | s_0 = s, a_i = \pi(s_i)].$$

Bellman's Equations ($\forall s \in S$):

$$\boxed{V^{\pi}(s) = \sum_{s' \in S} T(s, \pi(s), s') \left[R(s, \pi(s), s') + \gamma V^{\pi}(s')\right].}$$

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Define ($\forall s \in S, \forall a \in A$):

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Thus, given S, A, T, R, γ, and a fixed policy π, we can solve Bellman's equations efficiently to obtain, $\forall s \in S, \forall a \in A$, $V^{\pi}(s)$ and $Q^{\pi}(s, a)$.

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It can be shown that there exists a policy $\pi^* \in \Pi$ such that

$$\forall \pi \in \Pi \, \forall s \in S: \, V^{\pi^*}(s) \geq V^{\pi}(s).$$

V^{π^*} is denoted V^*, and Q^{π^*} is denoted Q^*.

There could be multiple optimal policies π^*, but V^* and Q^* are unique.

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Learning problem:

Given S, A, γ, and the facility to follow a trajectory by sampling from T and R, how can we find an optimal policy π^*? We need to be sample-efficient.

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Another way. We can apply the policy iteration algorithm, which is typically more efficient in practice.

- Pick an initial policy π arbitrarily.
- Compute Q^π using Bellman's equations.
- converged $\leftarrow$ `false`.
- Repeat
  - Set π' as: $\forall s \in S$, $\pi'(s) = \text{argmax}_a\, Q^\pi(s, a)$ (break ties arbitrarily).[Improvement]
  - Compute $Q^{\pi'}$ using Bellman's equations. [Evaluation]
  - If ($Q^{\pi'} = Q^\pi$), converged $\leftarrow$ `true`.
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- Until converged.
- Return π (which is provably optimal).

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Other ways. Value iteration and its various "mixtures" with policy iteration.

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Learning

Given S, A, γ, and the facility to follow a trajectory by sampling from T and R, how can we find an optimal policy π^*?

Various classes of learning methods exist. We will consider a simple one called Q-learning, which is a temporal difference learning algorithm.

- Let Q be our "guess" of Q^*: for every state s and action a, initialise $Q(s, a)$ arbitrarily. We will start in some state s_0.
- For $t = 0, 1, 2, \ldots$
  - Take an action a_t, chosen uniformly at random with probability ϵ, and to be $\text{argmax}_a\, Q(s_t, a)$ with probability $1 - \epsilon$.
  - The environment will generate next state s_{t+1} and reward r_{t+1}.
  - Update: $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha_t(r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t))$.

  [ϵ: parameter for "ϵ-greedy" exploration] [α_t: learning rate]
  [$r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t)$: temporal difference prediction error]

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  [ϵ: parameter for "ϵ-greedy" exploration] [α_t: learning rate]
  [$r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t)$: temporal difference prediction error]

For $\epsilon \in (0, 1]$ and $\alpha_t = \frac{1}{t}$, it can be proven that as $t \to \infty$, $Q \to Q^*$.

Q-Learning

Christopher J. C. H. Watkins and Peter Dayan. Machine Learning, 1992.

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Practical Implementation and Evaluation of Learning Algorithms

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Generalized Model Learning for Reinforcement Learning on a Humanoid Robot

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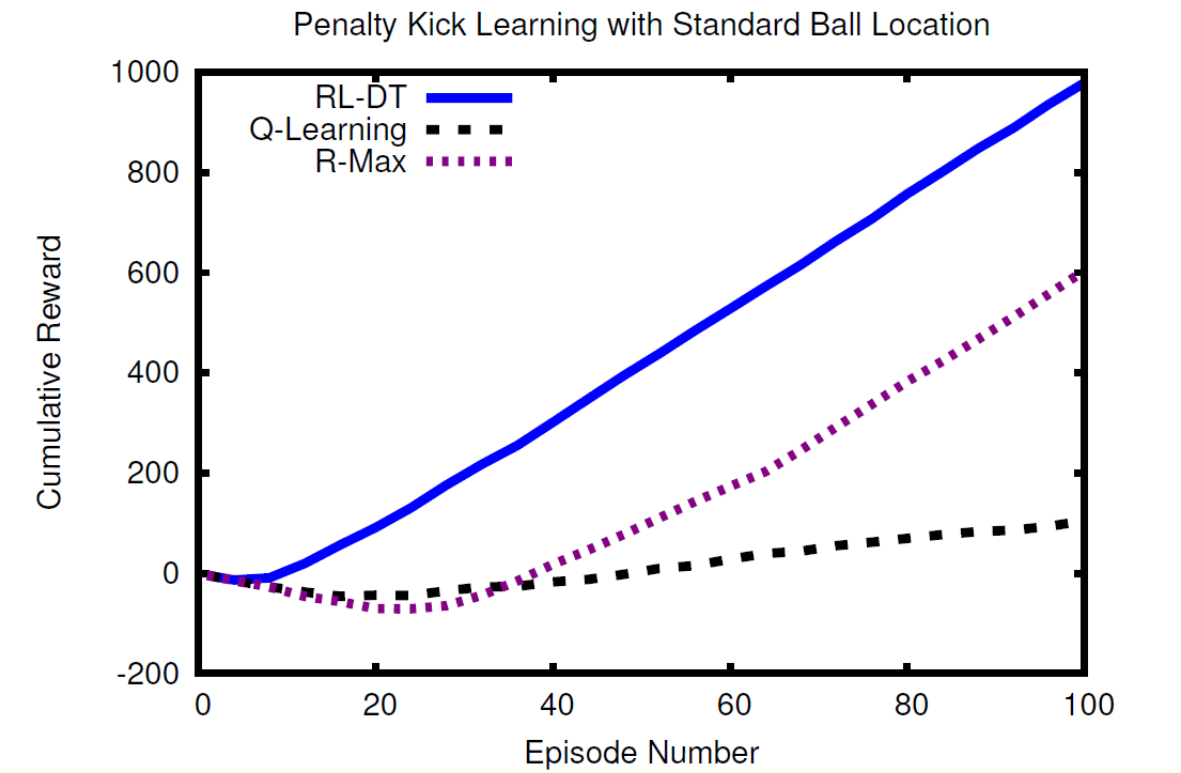

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Research Challenges

- Exploration
- Generalisation (over states and actions)
- State aliasing (partial observability)
- Multiple agents, nonstationary rewards and transitions
- Abstraction (over states and over time)
- Proofs of convergence, sample-complexity bounds

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My thesis question:

"How well do different learning methods for sequential decision making perform in the presence of state aliasing and generalization; can we develop methods that are both sample-efficient and capable of achieving high asymptotic performance in their presence?"

Learning Methods for Sequential Decision Making with Imperfect Representations
Shivaram Kalyanakrishnan. Ph.D. dissertation, University of Texas at Austin, 2011.

Practice $\Longrightarrow$ Imperfect Representations

Task	State Aliasing	State Space	Policy Representation (Number of features)
Backgammon (T1992)	Absent	Discrete	Neural network (198)
Job-shop scheduling (ZD1995)	Absent	Discrete	Neural network (20)
Tetris (BT1906)	Absent	Discrete	Linear (22)
Elevator dispatching (CB1996)	Present	Continuous	Neural network (46)
Acrobot control (S1996)	Absent	Continuous	Tile coding (4)
Dynamic channel allocation (SB1997)	Absent	Discrete	Linear (100's)
Active guidance of finless rocket (GM2003)	Present	Continuous	Neural network (14)
Fast quadrupedal locomotion (KS2004)	Present	Continuous	Parameterized policy (12)
Robot sensing strategy (KF2004)	Present	Continuous	Linear (36)
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Dynamic bipedal locomotion (TZS2004)	Present	Continuous	Feedback control policy (2)
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Robot soccer keepaway (SSK2005)	Present	Continuous	Tile coding (13)
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Optimized trade execution (NFK2007)	Present	Discrete	Tabular (2-5)
Blimp control (RPHB2007)	Present	Continuous	Gaussian Process (2)
9 $\times$ 9 Go (SSM2007)	Absent	Discrete	Linear ($\approx$1.5 million)
Ms. Pac-Man (SL2007)	Absent	Discrete	Rule list (10)
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Soccer opponent "hassling" (GRT2009)	Present	Continuous	Neural network (9)
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Perfect representations (fully observable, enumerable states) are impractical.

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Thank you!

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