Instructions

- Answer the questions in the space provided in this sheet. If you run out of space, affix extra pages at the end. Clearly indicate question numbers and your roll number on each of the extra pages.
- You are encouraged to print the homework on both sides of a page (duplex mode).
- Do not in any event copy solutions from internet, friends, or any other source.

1. (2 points) Consider the following syntactic classes of formulae:

   - $\varphi := p \mid \neg \varphi \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid \varphi \to \varphi$
   - $\psi := p \mid \neg \psi \mid \psi \land \psi$

   Define a transformation $T$ from formulae in $\varphi$ to equivalent formulae in $\psi$. The transformations are to be defined inductively on the syntax of $\varphi$. For instance, $T(\varphi_1 \land \varphi_2) = T(\varphi_1) \land T(\varphi_2)$. Give similar rules for other syntactic constructs of $\varphi$.

2. (3 points) Let $\varphi = p \land q \leftrightarrow r$ and $\psi = (q \to \neg r) \lor p$ Draw BDDs of $\varphi$ and $\psi$ (you need not show the construction). Construct the BDD of $\neg \varphi \land \psi$ using BDDs of $\varphi$ and $\psi$ and show all the steps of this construction. Use the variable ordering $p < q < r$ (from top-to-bottom) for all BDDs.
3. Consider the formula \( \varphi = (p \rightarrow q \land r \rightarrow s) \land \neg(p \land r \rightarrow q \land s) \).

(a) (3 points) Check satisfiability of \( \varphi \) using the DPLL-based procedure studied in the class. Clearly indicate the decision, implication graph (with edges annotated by clause numbers), learned clause, and backtracking level at every step.

(b) (2 points) In part (a) above, if you end up proving that \( \varphi \) is unsat, then give the resolution graph and work out resolution proof of unsat. Otherwise, give a satisfying assignment to \( \varphi \).

---

**Evaluation**

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points:</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Score:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>