Instructions

- Answer the questions in the space provided in this sheet. If you run out of space, affix extra pages at the end. Clearly indicate question numbers and your roll number on each of the extra pages.
- You are encouraged to print the homework on both sides of a page (duplex mode).
- Do not in any event copy solutions from internet, friends, or any other source.

1. (2 points) Prove by resolution that \((q \rightarrow r) \rightarrow (r \rightarrow p)\) is a valid formula.

2. (2 points) Prove that no formula in Tseitin’s encoding can be a valid formula. Consider only non-trivial formulae, that is, except the propositional constants like true and false.
3. (1 point) Let $f$ be the BDD of a formula $\varphi$. Give an algorithm using the APPLY function for constructing the BDD $f'$ of negation of the formula $\varphi$.

4. Consider the following clauses:
   
   $c_1 = \neg p \lor q \lor t$, $c_2 = \neg p \lor \neg t$, $c_3 = r \lor s$, $c_4 = \neg r \lor s$, and $c_5 = \neg q \lor \neg s$.

   (a) (1½ points) With $p$ as the root, draw the implication graph.

   (b) (1½ points) If it is a conflicting graph then derive all the asserting clauses using resolution and show the derivations clearly. Otherwise, give a satisfying assignment to the conjunction of the clauses.
5. (2 points) Construct the binary decision diagram compositionally (bottom-up) for \( a \lor b \land c \leftrightarrow d \) with the top-to-bottom variable order \( a < b < c < d \). Show all the steps clearly.