Equivalence of CTL and LTL formulas
Let $\Phi$ be a $\textbf{CTL}$ formula and $\varphi$ an $\textbf{LTL}$ formula.
Let $\Phi$ be a $\textbf{CTL}$ formula and $\varphi$ an $\textbf{LTL}$ formula.

\[ \Phi \equiv \varphi \iff \text{for all transition systems } \mathcal{I} \text{ and all states } s \text{ in } \mathcal{I}: \]

\[ s \models_{\text{CTL}} \Phi \iff s \models_{\text{LTL}} \varphi \]
Let $\Phi$ be a $\textbf{CTL}$ formula and $\varphi$ an $\textbf{LTL}$ formula.

$\Phi \equiv \varphi$ iff for all transition systems $\mathcal{T}$ and all states $s$ in $\mathcal{T}$:

$$s \models_{\text{CTL}} \Phi \iff s \models_{\text{LTL}} \varphi$$

e.g.,

<table>
<thead>
<tr>
<th>CTL formula $\Phi$</th>
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<tbody>
<tr>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td>$\forall a$</td>
<td>$\Diamond a$</td>
</tr>
<tr>
<td>$\forall (a \cup b)$</td>
<td>$a \cup b$</td>
</tr>
</tbody>
</table>

$a, b \in \text{AP}$
## More examples

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</tr>
<tr>
<td>$\forall (a \lor b)$</td>
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<tr>
<td>$\forall \Box a$</td>
<td>$\Box a$</td>
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<tr>
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infinitely often $a$
## More examples

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</tr>
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<td>$\forall \Box \Diamond a$</td>
<td>$\Box \Diamond a$</td>
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but: $\forall \Diamond \forall \Box a \not\equiv \Diamond \Box a$

infinitely often $a$
The CTL formula $\forall \diamond \forall a$
The CTL formula $\forall \diamond \forall \Box a$

$s \models \forall \diamond \forall \Box a$ iff on each path $\pi$ from $s$ there is a state $t$ with $t \models \forall \Box a$
The CTL formula $\forall \diamond \forall \Box a$

$s \models \forall \diamond \forall \Box a$ iff on each path $\pi$ from $s$

there is a state $t$ with $t \models \forall \Box a$

i.e., all states in the computation tree of $t$ fulfill $a$
The CTL formula $\forall \Diamond \forall \square a$

$s \models \forall \Diamond \forall \square a$ iff on each path $\pi$ from $s$
there is a state $t$ with $t \models \forall \square a$

i.e., all states in the computation tree of $t$ fulfill $a$
The CTL formula $\forall \lozenge \forall \square a$

$s \models \forall \lozenge \forall \square a$ iff on each path $\pi$ from $s$
there is a state $t$ with $t \models \forall \square a$

i.e., all states in the computation tree of $t$ fulfill $a$
TheCTL formula $\forall\lozenge\forall\square a$

$s \models \forall\lozenge\forall\square a$ iff on each path $\pi$ from $s$
there is a state $t$ with $t \models \forall\square a$

i.e., all states in the computation tree of $t$ fulfill $a$
◊□a ⊭ □◊A□a
\[ \Diamond \Box a \not\equiv \forall \forall \Box a \]

To prove that

\[ \forall \forall \Box a \not\equiv \Diamond \Box a \]

we provide an example for a TS \( \mathcal{T} \) s.t.

\[ \begin{align*}
\mathcal{T} & \models_{\text{LTL}} \Diamond \Box a \\
\mathcal{T} & \not\models_{\text{CTL}} \forall \forall \Box a
\end{align*} \]
transition system $\mathcal{I}$
transition system $\mathcal{T}$

$\mathcal{T} \models_{LTL} \Diamond \Box a$
\[ \Diamond \Box a \neq \forall \forall \forall \Box a \]

transition system \( T \)

\[ T \models_{\text{LTL}} \Diamond \Box a \]

\[ T \not\models_{\text{CTL}} \forall \forall \forall \Box a \]

computation tree
\( \Diamond \Box a \not\equiv \forall \forall \Box a \)

transition system \( \mathcal{T} \)

\[
\begin{align*}
\{a\} & \quad \emptyset & \quad \{a\} \\
\end{align*}
\]

\( \mathcal{T} \models_{\text{LTL}} \Diamond \Box a \)

\( \mathcal{T} \not\models_{\text{CTL}} \forall \Diamond \forall \Box a \)

\( \text{Sat}(\forall \Box a) = \{\bullet\} \)

computation tree
From CTL to LTL, if possible
For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent LTL formula
- or ...
From CTL to LTL, if possible

For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers $\exists$ and $\forall$

*without proof*
From CTL to LTL, if possible

For each **CTL** formula Φ the following holds:

- either there is no equivalent LTL formula
- or Φ ≡ ϕ

where ϕ is the **LTL** formula obtained from Φ by removing of all path quantifiers ∃ and ∀

\[ Φ = ∀∃∀□a \]
For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent **LTL** formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing all path quantifiers $\exists$ and $\forall$

\[
\Phi = \forall \diamond \forall \Box a \\
\downarrow \\
\varphi = \diamond \Box a
\]
From CTL to LTL, if possible

For each **CTL formula** \( \Phi \) the following holds:

- either there is **no** equivalent LTL formula
- or \( \Phi \equiv \varphi \)

where \( \varphi \) is the **LTL formula** obtained from \( \Phi \) by removing of all path quantifiers \( \exists \) and \( \forall \)

\[ \Phi = \forall \diamond \forall \Box a \]
\[ \downarrow \]
\[ \varphi = \diamond \Box a \neq \Phi \]
From CTL to LTL, if possible

For each \textbf{CTL} formula \( \Phi \) the following holds:

- either there is \textbf{no} equivalent \textbf{LTL} formula
- or \( \Phi \equiv \psi \)

where \( \psi \) is the \textbf{LTL} formula obtained from \( \Phi \) by removing of all path quantifiers \( \exists \) and \( \forall \)

\( \Phi = \forall \diamond \forall \square a \)
\downarrow
\psi = \diamond \square a \not\equiv \Phi \\

\textit{hence:} there is no LTL formula equivalent to \( \Phi \)
From CTL to LTL, if possible

For each **CTL** formula \( \Phi \) the following holds:

- either there is **no** equivalent LTL formula
- or \( \Phi \equiv \varphi \)

where \( \varphi \) is the **LTL** formula obtained from \( \Phi \) by removing of all path quantifiers \( \exists \) and \( \forall \)

\[ \Phi = \forall \square \forall \Diamond a \]

*without proof*
From CTL to LTL, if possible

For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers $\exists$ and $\forall$

\[ \Phi = \forall \Box \forall \Diamond a \]

\[ \downarrow \]

\[ \varphi = \Box \Diamond a \]

*without proof*
From CTL to LTL, if possible

For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers $\exists$ and $\forall$

\[ \Phi = \forall \Box \forall \Diamond a \]
\[ \downarrow \]
\[ \varphi = \Box \Diamond a \equiv \Phi \]

“infinitely often $a$”
From CTL to LTL, if possible

For each **CTL** formula $\Phi$ the following holds:

- either there is no equivalent LTL formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers $\exists$ and $\forall$

$$\Phi = \forall \Diamond (a \land \forall \bigcirc a)$$
For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers $\exists$ and $\forall$

### Example

\[
\Phi = \forall \lozenge (a \land \forall \lozenge a)
\]

\[
\downarrow
\]

\[
\varphi = \Box (a \land \lozenge a)
\]
For each **CTL** formula $\Phi$ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers $\exists$ and $\forall$

Without proof

$$
\Phi = \forall \Box (a \land \forall \Diamond a) \\
\Downarrow \\
\varphi = \Box (a \land \Diamond a) \not\equiv \Phi
$$
For each **CTL** formula \( \Phi \) the following holds:

- either there is **no** equivalent LTL formula
- or \( \Phi \equiv \varphi \)

where \( \varphi \) is the **LTL** formula obtained from \( \Phi \) by removing of all path quantifiers \( \exists \) and \( \forall \)

\[
\Phi = \forall \lozenge (a \land \forall \lozenge a) \\
\downarrow \\
\varphi = \lozenge (a \land \lozenge a) \not\equiv \Phi
\]

**hence**: there is no LTL formula equivalent to \( \Phi \)
◊(a \land \Box a) \neq \Box \Box (a \land \Box \Box a)
\( (a \land \bigcirc a) \neq \forall \diamond (a \land \forall \bigcirc a) \)

To prove that

\( (a \land \bigcirc a) \neq \forall \diamond (a \land \forall \bigcirc a) \)

we provide an example for a TS \( \mathcal{T} \) s.t.

\[ \mathcal{T} \models_{\text{LTL}} \diamond (a \land \bigcirc a) \]

\[ \mathcal{T} \not\models_{\text{CTL}} \forall \diamond (a \land \forall \bigcirc a) \]
\( \Diamond (a \land \Box a) \neq \forall \Diamond (a \land \forall \Box a) \)
\( \Diamond (a \land \Box a) \not\equiv \forall \Diamond (a \land \forall \Box a) \)

\( T \models_{\text{LTL}} \Diamond (a \land \Box a) \)

\[ = \emptyset \]

\[ = \{a\} \]
\(\Diamond (a \land \Box a) \neq \forall \Diamond (a \land \forall \Box a)\)

\[\mathcal{T} \models_{\text{LTL}} \Diamond (a \land \Box a)\]

\[\text{trace}(s_0 s_1 s_2^\omega) = \{a\} \{a\} \emptyset^\omega\]

\[\text{trace}(s_0 s_3 s_4^\omega) = \{a\} \emptyset \{a\}^\omega\]
\( \Diamond (a \land \Box a) \not\equiv \forall \Diamond (a \land \forall \Box a) \)

\[
\begin{align*}
\mathcal{T} &\models_{\text{LTL}} \Diamond (a \land \Box a) \\
\mathcal{T} &\not\models_{\text{CTL}} \forall \Diamond (a \land \forall \Box a)
\end{align*}
\]

\[
\begin{align*}
\text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\
\text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega
\end{align*}
\]

\[
\begin{align*}
\circ &= \emptyset \\
\bullet &= \{a\}
\end{align*}
\]
\( \lozenge (a \land \Box a) \not\equiv \forall \lozenge (a \land \forall \Box a) \)
\(\Diamond (a \land \Box a) \neq \forall \Box (a \land \forall \Box a)\)
The expressive powers of \textbf{LTL} and \textbf{CTL} are incomparable.
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall \Diamond (a \land \forall \Box a)$, $\forall \Diamond \forall \Box a$ and $\forall \Box \exists \Diamond a$ have no equivalent LTL formula.
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall \diamond (a \land \forall \bigcirc a)$, $\forall \diamond \forall \square a$ and $\forall \bigcirc \exists \diamond a$ have no equivalent LTL formula
- The **LTL** formula $\diamond \square a$ has no equivalent CTL formula
Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall \diamond (a \land \forall \lozenge a)$, $\forall \diamond \forall \square a$ and $\forall \square \exists \diamond a$ have no equivalent LTL formula

- The **LTL** formula $\diamond \Box a$ has no equivalent CTL formula
Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall \diamond (a \land \forall \Box a)$, $\forall \diamond \forall \Box a$ and $\forall \Box \exists \diamond a$ have no equivalent LTL formula

- The **LTL** formula $\Box \diamond a$ has no equivalent CTL formula
The expressive powers of LTL and CTL are incomparable

- The CTL formulas $\forall \diamond (a \land \forall \lozenge a)$, $\forall \diamond \forall \square a$ and $\forall \square \exists \diamond a$ have no equivalent LTL formula.

- The LTL formula $\diamond \square a$ has no equivalent CTL formula.
Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall \Diamond (a \land \forall \Box a)$, $\forall \Diamond \forall \Box a$ and $\forall \Box \exists \Diamond a$ have no equivalent LTL formula

- The **LTL** formula $\Diamond \Box a$ has no equivalent CTL formula
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall \lozenge (a \land \forall \square a)$, $\forall \lozenge \forall \square a$ and $\forall \square \exists \lozenge a$ have no equivalent LTL formula

- The **LTL** formula $\lozenge \square a$ has no equivalent CTL formula
The **CTL** formulas

\[ \forall \Diamond ( a \land \forall \Box a ) \]
\[ \forall \Diamond \forall \Box a \]
\[ \forall \Box \exists \Diamond a \]

have no equivalent **LTL** formula
CTL properties that are not LTL-definable

The CTL formulas
\[ \forall \diamond (a \land \forall \Box a) \]
\[ \forall \diamond \forall \Box a \]
\[ \forall \Box \exists \diamond a \]

have no equivalent LTL formula

Proof uses the fact that for each CTL formula \( \Phi \):

- either there is no equivalent LTL formula
- or \( \Phi \equiv \varphi \) where \( \varphi \) is the LTL formula obtained from \( \Phi \) by removing of all path quantifiers
The **CTL** formulas

\[
\forall \diamond (a \land \forall \square a) \\
\forall \forall \forall \diamond a \\
\forall \forall \exists \diamond a
\]

have no equivalent **LTL** formula

**Proof** uses the fact that for each **CTL** formula $\Phi$:

- either there is **no** equivalent **LTL** formula
- or $\Phi \equiv \varphi$ where $\varphi$ is the **LTL** formula obtained from $\Phi$ by removing of all path quantifiers
### CTL properties that are not LTL-definable

The **CTL** formulas

\[
\forall \Diamond (a \land \forall \Box a) \\
\forall \Diamond \forall \Box a \\
\forall \Box \exists \Diamond a \leftarrow \text{alternative (direct) proof}
\]

have no equivalent **LTL** formula

**Proof** uses the fact that for each **CTL** formula \( \Phi \):

- either there is **no** equivalent **LTL** formula
- or \( \Phi \equiv \varphi \) where \( \varphi \) is the **LTL** formula obtained from \( \Phi \) by removing of all path quantifiers
There is no LTL formula equivalent to $\forall \Box \exists \Diamond a$. 
There is no LTL formula equivalent to $\forall \Box \exists \ Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \Box \exists \ Diamond a$
There is no LTL formula equivalent to $\forall \Box \text{ } \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \Box \exists \Diamond a$

consider the following TS $\mathcal{T}_1$:
There is no LTL formula equivalent to $\forall \square \exists \Diamond a$.

Suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \square \exists \Diamond a$.

Consider the following TS $\mathcal{T}_1$:

$$\text{Sat}(\exists \Diamond a) = \{ s, t \}$$
There is no LTL formula equivalent to $\forall \Box \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \Box \exists \Diamond a$

consider the following TS $\mathcal{T}_1$:

$$\varnothing \xrightarrow{a} \{a\}$$

$Sat(\exists \Diamond a) = \{s, t\}$

$\mathcal{T}_1 \models \forall \Box \exists \Diamond a$
There is no LTL formula equivalent to $\forall \square \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \square \exists \Diamond a$

consider the following TS $T_1$:

Suppose $\varphi$ is an LTL formula such that $\varphi \equiv \forall \square \exists \Diamond a$. Consider the following TS $T_1$:

$T_1 \models \forall \square \exists \Diamond a \implies T_1 \models \varphi$

$Sat(\exists \Diamond a) = \{s, t\}$

$T_1 \models \forall \square \exists \Diamond a \implies T_1 \models \varphi$
There is no LTL formula equivalent to $\forall \Box \exists \Diamond a$.

Suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \Box \exists \Diamond a$.

Consider the following TS $T_1$:

\[
\begin{array}{ccc}
\emptyset & \{a\} & \{s, t\} \\
\uparrow & \downarrow & \downarrow \\
s & t & \text{Sat}(\exists \Diamond a) = \{s, t\}
\end{array}
\]

Thus, $T_1 \models \forall \Box \exists \Diamond a \implies T_1 \models \varphi$.

Consider the following TS $T_2$:
There is no LTL formula equivalent to $\forall \square \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \square \exists \Diamond a$

consider the following TS $\mathcal{T}_1$:

```
∅       \{a\}       \{s, t\}
```

$\mathcal{T}_1 \models \forall \square \exists \Diamond a \implies \mathcal{T}_1 \models \varphi$

consider the following TS $\mathcal{T}_2$:

```
∅       \{∅\}
```

$Traces(\mathcal{T}_2) = \{∅^\omega\}$
There is no LTL formula equivalent to $\forall \square \exists \diamond a$.

Suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \square \exists \diamond a$.

Consider the following TS $\mathcal{T}_1$:

\[
\begin{align*}
\text{Sat}(\exists \diamond a) &= \{s, t\} \\
\mathcal{T}_1 \models \forall \square \exists \diamond a &\implies \mathcal{T}_1 \models \varphi
\end{align*}
\]

Consider the following TS $\mathcal{T}_2$:

\[
\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1)
\]
There is no LTL formula equivalent to $\forall \square \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \square \exists \Diamond a$

consider the following TS $T_1$:

$\varnothing \rightarrow \{a\}$

$T_1 \models \forall \square \exists \Diamond a \implies T_1 \models \varphi$

consider the following TS $T_2$:

$\varnothing \rightarrow \{\varnothing^\omega\}$

$\text{Traces}(T_2) = \{\varnothing^\omega\} \subseteq \text{Traces}(T_1) \subseteq \text{Words}(\varphi)$
There is no LTL formula equivalent to $\forall \Box \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \Box \exists \Diamond a$

consider the following TS $\mathcal{T}_1$:

\[
\begin{array}{c}
\emptyset \\
s \longrightarrow \{a\} \longrightarrow t
\end{array}
\]

$Sat(\exists \Diamond a) = \{s, t\}$

$\mathcal{T}_1 \models \forall \Box \exists \Diamond a \implies \mathcal{T}_1 \models \varphi$

consider the following TS $\mathcal{T}_2$:

\[
\begin{array}{c}
\emptyset \\
s \longrightarrow \emptyset \omega
\end{array}
\]

$Traces(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq Traces(\mathcal{T}_1) \subseteq Words(\varphi)$

Hence: $\mathcal{T}_2 \models \varphi$
There is no LTL formula equivalent to $\forall \Box \exists \Diamond a$

suppose $\varphi$ is an LTL formula s.t. $\varphi \equiv \forall \Box \exists \Diamond a$

consider the following TS $\mathcal{T}_1$:

![Diagram of TS $\mathcal{T}_1$]

$Sat(\exists \Diamond a) = \{s, t\}$

$\mathcal{T}_1 \models \forall \Box \exists \Diamond a \implies \mathcal{T}_1 \models \varphi$

consider the following TS $\mathcal{T}_2$:

![Diagram of TS $\mathcal{T}_2$]

$Traces(\mathcal{T}_2) = \{\varnothing^\omega\} \subseteq Traces(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$

Hence: $\mathcal{T}_2 \models \varphi$

$\implies \mathcal{T}_2 \models \forall \Box \exists \Diamond a$
There is no LTL formula equivalent to $\forall \square \exists \diamond a$.

Suppose $\varphi$ is an LTL formula such that $\varphi \equiv \forall \square \exists \diamond a$.

Consider the following TS $\mathcal{T}_1$:

$\varphi$ is equivalent to $\forall \square \exists \diamond a$.

$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$.

Consider the following TS $\mathcal{T}_2$:

$\mathcal{T}_2 \models \varphi$.

$\mathcal{T}_2 \models \forall \square \exists \diamond a \implies$ contradiction!!
The expressive powers of **LTL** and **CTL** are incomparable.

The **CTL** formulas $\forall \Diamond (a \land \forall \Box a)$, $\forall \Diamond \forall \Box a$ and $\forall \Box \exists \Diamond a$ have no equivalent **LTL** formula.

The **LTL** formula $\Box \Diamond a$ has no equivalent **CTL** formula.
The expressive powers of **LTL** and **CTL** are incomparable.

The **CTL** formulas $\forall \diamond(a \land \forall \lozenge a)$, $\forall \diamond \forall \Box a$ and $\forall \Box \exists \diamond a$ have no equivalent **LTL** formula.

The **LTL** formula $\diamond \Box a$ has no equivalent **CTL** formula.
There is no **CTL** formula which is equivalent to the **LTL** formula $\Diamond \Box a$
There is no CTL formula which is equivalent to the LTL formula $\lozenge \Box a$.

Proof (sketch): provide sequences $(\mathcal{I}_n)_{n \geq 0}$, $(\mathcal{I}'_n)_{n \geq 0}$ of transition systems such that for all $n \geq 0$:

1. $\mathcal{I}_n \not\models \lozenge \Box a$
2. $\mathcal{I}'_n \models \lozenge \Box a$
LTL formula $\Diamond \Box a$

There is no CTL formula which is equivalent to the LTL formula $\Diamond \Box a$

Proof (sketch): provide sequences $(\mathcal{T}_n)_{n \geq 0}$, $(\mathcal{T}'_n)_{n \geq 0}$ of transition systems such that for all $n \geq 0$:

1. $\mathcal{T}_n \not\models \Diamond \Box a$
2. $\mathcal{T}'_n \models \Diamond \Box a$
3. $\mathcal{T}_n$ and $\mathcal{T}'_n$ satisfy the same CTL formulas length $\leq n$
Transition systems $\mathcal{I}_n$ and $\mathcal{I}_n'$
Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$
Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$
Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$
Transition systems $\mathcal{I}_n$ and $\mathcal{I}_n'$
Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$
Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$
Transition systems $\mathcal{I}_n$ and $\mathcal{I}'_n$

$\mathcal{I}_n$ and $\mathcal{I}'_n$ comparison

$\mathcal{I}_n \not\models \Box \Box a$

$\mathcal{I}'_n \models \Box \Box a$
Transition systems $\mathcal{I}_n$ and $\mathcal{I}_n'$

For all $\textbf{CTL}$ formulas $\Phi$ of length $|\Phi| \leq n$:

$\begin{align*}
    s_n &\models \Phi \iff s_n' \models \Phi \\
    t_n &\models \Phi \iff t_n' \models \Phi
\end{align*}$
Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

For all $\text{CTL}$ formulas $\Phi$ of length $|\Phi| \leq n$:

$s_n \models \Phi$ iff $s'_n \models \Phi$

$t_n \models \Phi$ iff $t'_n \models \Phi$

Hence: $\mathcal{T}_n$ and $\mathcal{T}'_n$ fulfill the same $\text{CTL}$ formulas of length $\leq n$
Does $\forall \lozenge (a \land \exists \lozenge a) \equiv \lozenge (a \land \lozenge a)$ hold?
Does \( \forall \square (a \land \exists \Diamond a) \equiv \Diamond (a \land \Diamond a) \) hold?

answer: no.
CTL vs LTL

Does $\forall \Box (a \land \exists \Diamond a) \equiv \Diamond (a \land \Diamond a)$ hold?

answer: no.

\[ T \]

$T = \{a\}$

$O = \emptyset$
CTL vs LTL

Does $\forall \diamond (a \land \exists \circ a) \equiv \diamond (a \land \circ a)$ hold?

answer: no.

$\mathcal{T} \not\models \diamond (a \land \circ a)$
CTL vs LTL

Does $\forall \Diamond (a \land \exists \bigcirc a) \equiv \Diamond (a \land \bigcirc a)$ hold?

answer: no.

$\mathcal{T}$

\[ \begin{array}{c}
\mathcal{T} \\
 S_0 \\
 S_1 \\
 S_2
\end{array} \]

$\mathcal{T} \not\models \Diamond (a \land \bigcirc a)$

note: $\pi = s_0 s_2 s_2 s_2 \ldots$ is a path in $\mathcal{T}$ with

$\text{trace}(\pi) = \{a\} \emptyset \emptyset \emptyset \ldots \notin \text{Words}(\Diamond (a \land \bigcirc a))$
Does $\forall \lozenge(a \land \exists \diamond a) \equiv \lozenge(a \land \Diamond a)$ hold?

answer: no.

$\mathcal{T} \not\models \lozenge(a \land \Diamond a)$

$\mathcal{T} \models \forall \lozenge(a \land \exists \diamond a)$
Does $\forall \Diamond (a \land \exists \bigcirc a) \equiv \Diamond (a \land \circ a)$ hold?

**answer:** no.

\[ T \not\models \Diamond (a \land \circ a) \]

\[ T \models \forall \Diamond (a \land \exists \bigcirc a) \]

\[ \text{Sat}(\exists \bigcirc a) = \{s_0, s_1\} \]

\[ \text{Sat}(\forall \Diamond (a \land \exists \bigcirc a)) = \{s_0, s_1\} \]
For each NBA $A$ there is a CTL formula $\Phi$ such that for all transition systems $T$:

$$T \models \Phi \iff \text{Traces}(T) \subseteq L_\omega(A)$$
For each NBA $\mathcal{A}$ there is a CTL formula $\Phi$ such that for all transition systems $\mathcal{T}$:

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

wrong.
Correct or wrong?

For each NBA $\mathcal{A}$ there is a CTL formula $\Phi$ such that for all transition systems $\mathcal{T}$:

$$\mathcal{T} \models \Phi \iff \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

Wrong. Consider, e.g., an NBA $\mathcal{A}$ with

$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Diamond \Box a)$$
Correct or wrong?

For each NBA $A$ there is a CTL formula $\Phi$ such that for all transition systems $T$:

$$T \models \Phi \iff \text{Traces}(T) \subseteq L_\omega(A)$$

**Wrong.** Consider, e.g., an NBA $A$ with

$$L_\omega(A) = \text{Words}(\Diamond \Box a)$$

But there is no CTL formula $\Phi$ such that $\Phi \equiv \Diamond \Box a$
If $\Phi$ is **CTL** formula and $\varphi$ an **LTL** formula such that $\Phi \equiv \varphi$ then $\neg \Phi \equiv \neg \varphi$
Correct or wrong?

If $\Phi$ is $\textbf{CTL}$ formula and $\varphi$ an $\textbf{LTL}$ formula such that $\Phi \equiv \varphi$ then $\neg \Phi \equiv \neg \varphi$

wrong.
If $\phi$ is **CTL** formula and $\varphi$ an **LTL** formula such that $\phi \equiv \varphi$ then $\neg \phi \equiv \neg \varphi$

**Wrong.** E.g.,

$$\phi = \forall \square \forall \diamond a, \quad \varphi = \square \diamond a$$
If $\Phi$ is CTL formula and $\varphi$ an LTL formula such that $\Phi \equiv \varphi$ then $\neg \Phi \equiv \neg \varphi$

Wrong. E.g.,

$$\Phi = \forall \Box \forall \Diamond a, \quad \varphi = \Box \Diamond a$$

1. $\Phi \equiv \varphi$
If $\Phi$ is CTL formula and $\varphi$ an LTL formula such that $\Phi \equiv \varphi$ then $\neg \Phi \equiv \neg \varphi$

Wrong. E.g.,

$$\Phi = \forall \square \forall \Diamond a, \quad \varphi = \square \Diamond a$$

- $\Phi \equiv \varphi$
- there is no CTL formula that is equivalent to $\neg \varphi \equiv \Diamond \Box \neg a$
Correct or wrong?

\[ s \models \exists \Box \Diamond a \quad \text{iff} \quad \text{there is a path } \pi \in Paths(s) \text{ with } \pi \models \Box \Diamond a \]
Correct or wrong?

$s \models \exists \Box \Diamond a$ iff there is a path $\pi \in Paths(s)$ with $\pi \models \Box \Diamond a$

wrong.
Correct or wrong?

\[ s \models \exists \Box \Diamond a \quad \text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

wrong.

\begin{equation}
\end{equation}
Correct or wrong?

$s \models \exists \Box \Diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \Box \Diamond a$

wrong.
Correct or wrong?

\[ s \models \exists \Box \Diamond a \quad \text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

\text{wrong.}

\text{note that: } s \models \exists \Diamond a
Correct or wrong?

$s \models \exists \Box \Diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \Box a$

wrong.

![Diagram]

note that: $s \models \exists \Diamond a$

thus: $s s s s \ldots \models \Box \exists \Diamond a$
Correct or wrong?

$s \models \exists \Diamond \Diamond a$ iff there is a path $\pi \in \mathit{Paths}(s)$ with $\pi \models \Box \Diamond a$

wrong.

note that: $s \models \exists \Diamond a$

thus: $s s s s \ldots \models \Box \exists \Diamond a$

but there is no path where $\Box \Diamond a$ holds
Correct or wrong?

\[ s \models \exists \Box \Diamond a \quad \text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

wrong.

\[ s \models \exists \Diamond \Box a \quad \text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Diamond \Box a \]
Correct or wrong?

\[ s \models \exists \Box \Diamond a \iff \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

wrong.

\[ s \models \exists \Diamond \Box a \iff \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Diamond \Box a \]

correct.
Correct or wrong?

$s \models \exists \Box \Diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \Box \Diamond a$

Wrong.

$s \models \exists \Diamond \exists \Box a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \Diamond \Box a$

Correct. $\exists \Diamond \exists \Box a \equiv \neg \forall \forall \Diamond \neg a$
Correct or wrong?

\[
\begin{align*}
s \models \exists \Diamond \Box a \quad &\text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \\
\end{align*}
\]

Wrong.

\[
\begin{align*}
s \models \exists \Diamond \exists \Box a \quad &\text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \\
\pi \models \Diamond \Box a
\end{align*}
\]

Correct. \[\exists \Diamond \exists \Box a \equiv \neg \forall \Box \forall \neg a\] 

\[s \models \exists \Diamond \exists \Box a\]
Correct or wrong?

\[ s \models \exists \Box \Diamond a \quad \text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

wrong.

\[ s \models \exists \Diamond \Box a \quad \text{iff} \quad \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Diamond \Box a \]

correct.

\[ \exists \Diamond \Box a \equiv \neg \forall \Box \Diamond \neg a \]

\[ s \models \exists \Diamond \Box a \quad \text{iff} \quad s \not\models \forall \Box \Diamond \neg a \]
Correct or wrong?

\[ s \models \exists \Box \Diamond a \text{ iff there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

Wrong.

\[ s \models \exists \Diamond \Box a \text{ iff there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Diamond \Box a \]

Correct.

\[ \exists \Diamond \Box a \quad \equiv \quad \neg \forall \Box \Diamond \neg a \]

\[ s \models \exists \Diamond \Box a \text{ iff } s \not\models \forall \Box \Diamond \neg a \]

iff \[ s \not\models \Box \Diamond \neg a \]
Correct or wrong?

\[ s \models \exists \lozenge \lozenge a \text{ iff there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \lozenge a \]

wrong.

\[ s \models \exists \lozenge \Box a \text{ iff there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \lozenge \Box a \]

correct.

\[ \exists \lozenge \Box a \equiv \neg \forall \lozenge \lozenge \neg a \]

\[ s \models \exists \lozenge \Box a \text{ iff } s \not\models \forall \lozenge \lozenge \neg a \]

iff \[ s \not\models \Box \lozenge \neg a \equiv \neg \lozenge \Box a \]
Correct or wrong?

\[ s \models \exists \Box \Diamond a \iff \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Box \Diamond a \]

Wrong.

\[ s \models \exists \Diamond \Box a \iff \text{there is a path } \pi \in \text{Paths}(s) \text{ with } \pi \models \Diamond \Box a \]

Correct.

\[ \exists \Diamond \Box a \equiv \neg \forall \Box \Diamond \neg a \]

\[ s \models \exists \Diamond \Box a \iff s \not\models \forall \Box \Diamond \neg a \]

iff \[ s \not\models \Box \Diamond \neg a \equiv \neg \Diamond \Box a \]

iff there is a path \( \pi \) ....
Correct or wrong?

There is an LTL formula $\phi$ with $\phi \equiv \neg\Diamond\Diamond a$
Correct or wrong?

There is an LTL formula $\varphi$ with $\varphi \equiv \neg\exists\Diamond\exists\Diamond a$

correct
Correct or wrong?

There is an **LTL** formula $\varphi$ with $\varphi \equiv \neg \Box \Diamond \Diamond a$

**correct** as $\neg \Box \Diamond \Diamond a \equiv \Diamond \Diamond \neg a$
Correct or wrong?

There is an LTL formula $\varphi$ with $\varphi \equiv \neg \Diamond \Box a$

correct as $\neg \Diamond \Box a \equiv \forall \forall \Diamond \neg a \equiv \Box \Diamond \neg a$
Correct or wrong?

There is an LTL formula $\varphi$ with $\varphi \equiv \neg \exists \Box \exists \Box a$

correct as $\neg \exists \Box \exists \Box a \equiv \forall \Box \forall \neg \neg a \equiv \Box \Box \neg a$

$T \not\models \neg \exists \Box a$ iff there is a path $\pi \in Paths(T)$ with $\pi \models \Box a$
There is an LTL formula $\varphi$ with $\varphi \equiv \neg \exists \diamond \exists \Box a$.

correct as $\neg \exists \diamond \exists \Box a \equiv \forall \forall \forall \neg a \equiv \Box \Box \neg a$

$T \not\models \neg \exists \Box a$ iff there is a path $\pi \in \text{Paths}(T)$ with $\pi \models \Box a$

correct
Correct or wrong?

There is an **LTL** formula $\varphi$ with $\varphi \equiv \neg \exists \Diamond \exists \Box a$

**Correct** as $\neg \exists \Diamond \exists \Box a \equiv \forall \Box \forall \neg a \equiv \Box \Diamond \neg a$

$T \not \models \neg \exists \Box a$ iff there is a path $\pi \in \text{Paths}(T)$ with $\pi \models \Box a$

**Correct** $T \not \models \neg \exists \Box a$
Correct or wrong?

There is an **LTL** formula \( \varphi \) with \( \varphi \equiv \neg \Box \Diamond \Diamond a \)

**Correct** as \( \neg \Box \Diamond \Diamond a \equiv \forall \Box \forall \Diamond \neg a \equiv \Box \Diamond \neg a \)

\[ T \not\models \neg \Box a \text{ iff there is a path } \pi \in \text{Paths}(T) \text{ with } \pi \models \Box a \]

**Correct** \( T \not\models \neg \Box a \)

iff there is an initial state \( s \) with \( s \not\models \neg \Box a \)
There is an LTL formula $\varphi$ with $\varphi \equiv \neg \exists \Diamond \exists \Box a$

**Correct** as $\neg \exists \Diamond \exists \Box a \equiv \forall \Box \forall \neg a \equiv \Box \Box \neg a$

$T \not \models \neg \exists \Box a$ if there is a path $\pi \in \text{Paths}(T)$ with $\pi \models \Box a$

**Correct** $T \not \models \neg \exists \Box a$

iff there is an initial state $s$ with $s \not \models \neg \exists \Box a$

iff there is an initial state $s$ with $s \models \exists \Box a$
Correct or wrong?

There is an LTL formula \( \varphi \) with \( \varphi \equiv \neg \exists \lozenge \lozenge a \)

correct as \( \neg \exists \lozenge \lozenge a \equiv \forall \lozenge \forall \lozenge \neg a \equiv \lozenge \lozenge \neg a \)

\( T \not \models \neg \exists \lozenge a \) iff there is a path \( \pi \in \text{Paths}(T) \) with \( \pi \models \lozenge a \)

correct \( T \not \models \neg \exists \lozenge a \)

iff there is an initial state \( s \) with \( s \not \models \neg \exists \lozenge a \)

iff there is an initial state \( s \) with \( s \models \exists \lozenge a \)

iff there is a path \( \pi \in \text{Paths}(T) \) with \( \pi \models \lozenge a \)
There is an LTL formula $\varphi$ with $\varphi \equiv \neg\exists \Box \exists \Box a$

correct as $\neg\exists \Box \exists \Box a \equiv \forall \Box \forall \Box \neg a \equiv \Box \Box \neg a$

$T \not\models \neg\exists \varphi$ iff there is a path $\pi \in \text{Paths}(T)$ with $\pi \models \varphi$

correct $T \not\models \neg\exists \varphi$

iff there is an initial state $s$ with $s \not\models \neg\exists \varphi$

iff there is an initial state $s$ with $s \models \exists \varphi$

iff there is a path $\pi \in \text{Paths}(T)$ with $\pi \models \varphi$
Correct or wrong?

\[ \mathcal{I} \not\models \neg \forall \Box a \quad \text{iff} \quad \text{for all paths } \pi \in \text{Paths}(\mathcal{I}): \]

\[ \pi \models \Box a \]
Correct or wrong?

\[ T \not\models \neg \forall \Box a \iff \text{for all paths } \pi \in \text{Paths}(T): \]
\[ \pi \models \Box a \]

Wrong.
Correct or wrong?

\[ \mathcal{I} \not\models \neg \forall \Box a \iff \text{for all paths } \pi \in \text{Paths}(\mathcal{I}) : \pi \models \Box a \]

\[ \text{wrong.} \]

\[ \mathcal{I} \not\models \neg \forall \Box a \]
Correct or wrong?

\[ \mathcal{I} \not\models \neg \forall \square a \text{ iff for all paths } \pi \in \text{Paths}(\mathcal{I}): \]
\[ \pi \models \square a \]

wrong.

\[ \mathcal{I} \not\models \neg \forall \square a \]

iff there is an initial state \( s \) with \( s \not\models \neg \forall \square a \)
Correct or wrong?

\[ \mathcal{I} \not\models \neg \forall a \quad \text{iff} \quad \text{for all paths } \pi \in \text{Paths}(\mathcal{I}) : \quad \pi \models \Box a \]

Wrong.

\[ \mathcal{I} \not\models \neg \forall a \]

iff there is an initial state \( s \) with \( s \not\models \neg \forall a \)

iff there is an initial state \( s \) with \( s \models \forall a \)
Correct or wrong?

\[ \mathcal{I} \not\models \neg \forall \Box a \quad \text{iff for all paths } \pi \in \text{Paths}(\mathcal{I}) : \]
\[ \pi \models \Box a \]

Wrong.

\[ \mathcal{I} \not\models \neg \forall \Box a \]

iff there is an initial state \( s \) with \( s \not\models \neg \forall \Box a \)

iff there is an initial state \( s \) with \( s \models \forall \Box a \)

but there might be another initial state \( t \) s.t. \( t \not\models \forall \Box a \)
Correct or wrong?

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all CTL formulas $\Phi$: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$
If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all $\text{CTL}$ formulas $\Phi$: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

wrong.
Correct or wrong?

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all CTL formulas $\Phi$: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

Wrong.

$\mathcal{T}_1$: $\{a\} \rightarrow \{b\} \rightarrow \{b\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{b\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{a\}$

$\mathcal{T}_2$: $\{a\} \rightarrow \{b\} \rightarrow \{b\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{b\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{b\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{b\}$
Correct or wrong?

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent TS then for all CTL formulas $\Phi$: $\mathcal{T}_1 \models \Phi$ iff $\mathcal{T}_2 \models \Phi$

wrong.

$\mathcal{T}_1$: $\{a\}$ $\{b\}$

$\mathcal{T}_2$: $\{a\}$ $\{b\}$

$\mathcal{T}_1$ and $\mathcal{T}_2$ are trace equivalent
Correct or wrong?

If $T_1$ and $T_2$ are trace equivalent TS then for all CTL formulas $\Phi$: $T_1 \models \Phi$ iff $T_2 \models \Phi$

Wrong.

$T_1$: \{a\}, \{b\}

$T_2$: \{a\}, \{b\}

Consider the CTL formula:

$\Phi = \exists \Diamond a \land \exists \Diamond b$

$T_1 \not\models \Phi$

$T_2 \models \Phi$

$T_1$ and $T_2$ are trace equivalent