1. Give an example of a linear arithmetic formula ϕ that is satisfiable over rationals but not satisfiable over integers. ϕ should involve only integral constants.

2. Let n and m be relatively prime positive integers. What is the strongest atomic fact in the theory of uninterpreted functions that is implied by $y = F^n(y) \wedge y = F^m(y)$.

3. Give an example of a theory that is not stably infinite but which is convex. [In class, we saw an example of a theory that is not stably infinite. However, that theory was also not convex. Quite often, theories that are not stably infinite are also not convex, but not always.]

4. The purpose of this example is to demonstrate the importance of disjointedness condition required on theories T1 and T2 in the Nelson-Oppen combination methodology. Consider the following parity theory that shares the functions $\pm$ with integer linear arithmetic.

$$\text{Expressions } e \ := \ y \parallel c \parallel e_1 \pm e_2$$
$$\text{Atomic facts } g \ := \ \texttt{IsOdd}(e) \parallel \texttt{IsEven}(e)$$
$$\text{Axioms: } \forall e_1, e_2 : \texttt{IsOdd}(e_1) \wedge \texttt{IsEven}(e_2) \Rightarrow \texttt{IsOdd}(e_1 + e_2) \text{ and so on.}$$

   Give an example of a formula ϕ s.t.

   - ϕ is over combination of integer linear arithmetic and parity theory, i.e., ϕ uses the binary relation $\geq$ and unary relations `IsOdd`, `IsEven`.
   - ϕ is unsatiafiable.
   - Nelson-oppen combination methodology (in which we also share disjunction of equalities) would fail to identify unsatisfiability.

5. Write down the generic transfer function for logical abstract interpreter for the non-deterministic assignment $x :=?$ in terms of existential quantification. [In class, we saw that the generic transfer function for an assignment statement $x := e$ transforms the formula ϕ before the assignment into the formula $\exists x' : \phi[x'/x] \wedge (x = e[x'/x])$. Write a similar transformation for the non-deterministic assignment $x :=?$. Non-deterministic assignments are a commonly used abstraction for modeling those assignments in which the expression being assigned cannot be modeled by the underlying abstract domain.]

6. Consider the abstract domain of conjunctions of difference constraints. For this domain, write down the results of the following operations (as would be computed by the algorithm described in the class).

   - $\texttt{Join}(x = 0 \wedge y = 1,\ x = 4 \wedge y = 6)$
   - $\texttt{Eliminate}(x - y \leq 3 \wedge y - z \leq 4,\ y)$

7. Consider the abstract domain of conjunctions of equalities between uninterpreted function terms. For this domain, write down the results of the following operations (as would be computed by the algorithm described in the class).

   - $\texttt{Join}(y = F^4(y),\ y = F^6(y))$
   - $\texttt{Eliminate}(x = F^3(x) \wedge x = F^5(x) \wedge y = F^4(x),\ x)$