Automata Theory and Computability

Assignment 5

Due on Fri 29 November 2013.

1. Show that the following functions are computable by a Turing Machine in the sense discussed in class. Assume that the inputs are given in unary, with \(0^n\) representing the number \(n\). Give a complete description of the moves of the TM using the state diagram notation.

   (a) \(\text{square} : \mathbb{N} \rightarrow \mathbb{N}\) where \(\text{square}(n) = n^2\).

   (b) (integer division) \(\text{div} : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}\), where \(\text{div}(m, n)\) is the largest integer less than or equal to \(m/n\) if \(m \geq 0\) and 0 otherwise.

2. One of the following sets is r.e. and the other is not. Which is which?

   (a) \(\{M \mid L(M)\text{ contains at least 481 elements }\}\).

   (b) \(\{M \mid L(M)\text{ contains at most 481 elements }\}\).

3. Let \(L, K \subseteq \Sigma^*\). Define

   \[
   L/K = \{x \mid \exists y \in K, xy \in L\}
   \]

   (a) Show that if \(L\) is regular and \(K\) is any language, then \(L/K\) is regular.

   (b) Show that even if we are given a DFA for \(L\) and a Turing machine for \(K\), we cannot always construct an automaton for \(L/K\).

4. Show without using Rice’s theorem that neither the language

   \(\text{TOTAL} = \{M \mid M\text{ halts on all inputs}\}\)

   nor its complement is r.e.

5. Is it decidable to tell for a given CFG \(G\) whether the complement of \(L(G)\) is also a CFL? Justify your answer.

6. Show that it is undecidable to check whether the intersection of two DCFL’s is non-empty.

   \(\text{Hint:}\) represent a valid halting computation of \(M\) on \(x\) by a string of the form

   \[c_0\#\text{rev}(c_1)\#c_2\#\text{rev}(c_3)\#\cdots c_n\#\]

   as shown below, which is similar to the encoding we used for VALCOMPS\((M, x)\), except that each alternate configuration is reversed.
Show that this set of strings can be expressed as the intersection of two DCFL’s.

7. Is the language inclusion problem for deterministic pushdown automata (DPDA’s) (i.e. given DPDA’s $M_1$ and $M_2$, is $L(M_1) \subseteq L(M_2)$?) decidable? Justify your answer.